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Interregional redistribution and budget institutions with private information on intergenerational externality

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Abstract

We study a federal government's optimal redistributive policy across regions in the context of a model in which regions issue debt, invest in intergenerational public goods (IPGs), and have private information regarding the durability of their IPG investment. First, in both the complete-information and the asymmetric-information optimum, the region with a higher degree of intergenerational spillovers (H-region) should borrow more than the region with a lower degree (L-region). Second, to induce truth-telling under asymmetric information, the region not distorted on intertemporal allocation should be the contributor of redistribution. Third, the asymmetric-information optimum is implementable through decentralized regional debt decisions by imposing differentiated budget institutions: if H-region is distorted on intertemporal allocation, then it faces a debt floor; if L-region is distorted, then it faces a debt ceiling.

Keywords Interregional redistribution · Regional debt · Borrowing rules · Asymmetric information · Intergenerational public goods

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1 Introduction

As important components of modern fiscal institutions, both debt limits on local borrowing[1] and interregional redistribution via federal transfers[2] are implemented in many countries. Inspired by the empirical evidence from the U.S. and the European Union, Huber and Runkel (2008) present a theoretical argument that justifies the fiscal institution with lax budget rules for contributors and strict budget rules for recipients of federal transfers. They argue that this arrangement solves the self-selection problem the federal government faces in the presence of asymmetric information regarding the regional social discount factor. Even though they have provided an enlightening result, they just take into account the negative intergenerational externality induced by debt[3] while ignoring the positive intergenerational externality induced by the investment in intergenerational public goods (IPGs), such as basic science, public education, and environmental protection.[4] In this study we are interested in the joint effect of these two countervailing externalities on the optimal fiscal arrangement in a federation.

More specifically, we address the following questions. What are the optimal regional debt level and IPG investment that take into account the welfare of both present and future generations? If borrowing and spending decisions are decentralized to the regional governments, can we design interregional redistribution schemes and regional budget institutions such that the welfare optimum is implemented in the presence of regional heterogeneity and asymmetric information between the center and regions? While these questions are of considerable policy and theoretical interest, they have not been formally addressed, to the best of our knowledge.

We consider a two-generation, two-region economy under a fiscal federalism. These two regions are assumed to differ only in the extent of positive intergenerational spillovers generated by their respective IPGs. The degree of intergenerational spillovers, which may be interpreted as a measure of durability or quality of public goods, is the private information of each region. As usual, the benevolent federal government (or the mechanism designer) is responsible for interregional redistribution. We establish first the (constrained) optimal interregional redistribution and regional

[1] For example, the Australian federal government exercises control over state borrowing by subjecting all state borrowing to the approval of the Australian Loan Council (Mathews 1984). Also, Canada and many European countries, such as Switzerland, Germany and Austria, impose debt restrictions on local budgets (Bird and Slack 1983; Smekal 1984). In recent years, China's State Council announced a series of strict rules on how its local governments can issue and manage debt in order to control financial risks, moving to defuse local government debt bomb (e.g., The Economist 2015; Huang et al. 2018).

[2] See, e.g., the evidences reported by Mélitz and Zumer (2002) and The Economist (2016).

[3] Some theoretical arguments on this issue have been provided by economists. For example, Schultz and Sjöström (2004) show in a representative democracy with local elections that the median voters prefer shortsighted political leaders who will implement inefficiently high debt levels. Also, Dai et al. (2019) analyze a fiscal-policy game between two jurisdictions connected by mutual migrations and show that the first-best allocation can be achieved through Nash play by imposing the restriction that public consumption should be financed by a contemporary tax and not by borrowing.

[4] To distinguish from backward intergenerational goods, they are specifically called forward intergenerational goods in Rangel (2003).

debt policies that are incentive compatible for both regions. We then proceed to search for the federal redistribution scheme and appropriate regional budget institutions such that the asymmetric-information optimum can be truthfully implemented via regionally decentralized borrowing decisions.

The key message conveyed by our analysis is the following. First, in the welfare optimum the region with a higher degree of intergenerational spillovers (H-region henceforth) induced by its IPGs should be allocated with more public debt in the first generation than the region with a lower degree (L-region henceforth), regardless of whether these regions have private information on their degree of intergenerational spillovers or not. Second, in the asymmetric-information optimum, the region that is not distorted on intertemporal allocation and hence extracts the information rent should be the contributor of interregional redistribution, regardless of the type of the region under consideration.[5] In a region whose intertemporal allocation is not distorted, the intertemporal marginal rate of substitution between current and future public consumption is equal to the intertemporal rate of transformation. In particular, if IPG investment in L-region relative to that in H-region is above some threshold, then only the intertemporal allocation of L-region is not distorted. And third, to truthfully implement the asymmetric-information optimum with regions having autonomy in the choice of their public debt, differentiated budget institutions must be imposed: If H-region is distorted on intertemporal allocation, then it faces a debt floor; if, however, L-region is distorted, then it faces a debt ceiling.

The intuition for the first result above is straightforward. Since the responsibility to repay the debt plus interest is passed to the second generation, it is reasonable to allow the region with larger intergenerational spillovers to borrow more in the first generation. As more debt implies, ceteris paribus, more investment in IPGs by the first generation, the positive intergenerational externality generated by IPGs could (partially) offset the negative intergenerational externality generated by debt financing.

The intuition for the second result above is also easy to understand. Due to asymmetric information, the federal government as the principal who faces the rent extraction-allocative efficiency tradeoff is ready to accept some efficiency-reducing distortions in order to decrease the information rent extracted by the regions. As is common in the mechanism design literature, the top-type region is the only one that ends up making efficient decisions, reflected as non-distorted intertemporal allocation in the present context, and it must also provide a positive amount of transfers to the other region. In particular, we show that the top-type region does not necessarily correspond to the one with the largest degree of intergenerational spillovers. In what follows, we show that the indeterminacy of the top-type region in the asymmetric-information optimum mainly arises from our joint consideration of two opposite intergenerational externalities.

We just need to analyze why L-region might turn out to be the top-type region, which is equivalent to analyzing why it might have incentives to mimic H-region under asymmetric information. Recall first that the intertemporal rate of transformation is the rate at which savings in the first period can be transformed into consumption in the second period, and an increase in which implies an increase in the opportunity cost of borrowing. In the current model, as the positive intergenerational spillovers

5 This region's incentive-compatibility constraint must be binding in the welfare optimum.

induced by IPG investment partly offset the negative intergenerational externality caused by borrowing, the opportunity cost of borrowing is always smaller in H-region than in L-region, which is public knowledge. If L-region misreports its type, then it shall be allocated with a high level of debt in generation 1, which implies a high repayment of debt plus interest and hence a low level of consumption in generation 2. As such, the cost from cheating manifests as a reduction in the welfare of generation 2 because the true opportunity cost of borrowing in L-region is high rather than low. Nevertheless, such a reduction is somehow bounded because the high level of debt issued in generation 1 implies, ceteris paribus, a high level of IPG investment that increases the positive intergenerational spillovers, thus mitigating the distortion in its intertemporal allocation. In addition, the consumption and welfare in generation 1 may increase if the level of debt issued is larger than the positive amount of transfers paid to the other region. Importantly, depending on the underlying preference structure, the total benefit may outweigh the total cost from cheating, yielding a higher regional welfare than from telling the truth. This is indeed the case if its IPG investment is sufficiently high. The intuition is that a high IPG investment implies that the debt issued is larger than the transfers paid in generation 1, and also that the downward distortion placed on generation 2 is bounded. This thus explains why the relative productivity of the less productive region being above a well-defined lower bound suffices to identify the region as the top-type.

We now discuss the intuition for the third result above, namely the differentiated borrowing rules required for implementing the asymmetric-information optimum through decentralized regional debt decisions. If L-region is the top-type region whose intertemporal allocation is not distorted with respect to the first-best, the distortionary debt constraint is not desirable for this region. The fiscal constraint of a debt floor distorts the spending decision of H-region in favor of future public consumption, and hence the investment in IPGs, which makes explicit the implicit borrowing constraint contained in the asymmetric-information optimum. This level of debt floor renders the allocation of H-region unattractive for L-region who actually faces a higher opportunity cost of borrowing, so it voluntarily pays the lump-sum tax to the federal government instead of mimicking H-region. Specifically, the intertemporal spending decision of H-region in the asymmetric-information optimum is distorted such that the intertemporal marginal rate of substitution between current and future public consumption is smaller than the intertemporal rate of transformation. As such, the marginal utility from borrowing a dollar and spending it on first-period public consumption is smaller than the marginal utility from spending this dollar in the second period. This explains why solving the self-selection problem facing the center calls for a (distortionary) debt floor rather than a debt ceiling placed on the distorted H-region. The other case with H-region being the top-type region who is not distorted under asymmetric information can be analyzed similarly.

Since the seminal study of Oates (1972), the informational asymmetry between the federal government and geographically decentralized regions becomes the major justification of fiscal decentralization as well as the decentralization of local public goods provision in a federalism system. Indeed, there is a large number of studies that focus on examining the impact of asymmetric information on designing optimal interregional redistribution or interregional insurance mechanisms, such as Bucovetsky et al. (1998), Lockwood (1999), Bordignon et al. (2001), Cornes and Silva (2002), Breuillé and Gary-Bobo (2007), Huber and Runkel (2008), Kıbrıs and Tapkı (2014) and Dai et al. (2019a, b). However, none of these studies, except for Dai et al. (2019b), considers the current informational asymmetry regarding the durability of local IPGs. Interpreted as a measure of the quality of local IPGs, it is reasonable to assume that the durability of local IPGs is private information of local governments. We present an argument in favor of this assumption from the following two perspectives. Firstly, the quality of the physical output of some IPGs such as local environmental protection and R&D is objectively unobservable, at least in the short run, by the center who is in general not involved in the process of producing these public goods. Secondly, the local politicians have subjective incentives to hide/misreport such information for the sake of either getting more transfers, getting personal promotions, or avoiding punishments. For example, local politicians in China may get promoted to higher levels because of doing a good job in public infrastructure investment or establishing a business friendly environment, or may get punished for being responsible for *tofu-dreg projects*[6] in the provision of local IPGs, such as public schools, bridges and dams, that end up in very low quality or even tragedies.

To show how asymmetric information on intergenerational externality shapes the optimal policies in redistributing resources across heterogeneous regions, as well as in determining restrictions on regional debt levels, we compare our study with Huber and Runkel (2008) who considered the same policy issue in a similar context in which the local public goods are non-durable and the regional discount rate (instead of the durability of the public goods) is local governments' private information. They find that the recipient region of interregional redistribution should borrow less than the contributor region, and it must face a public debt ceiling so that welfare optimum can be implemented under regionally decentralized borrowing decisions. It turns out that their finding just represents one possible arrangement among others in the present context. In particular, this finding carries over to our study only when it is H-region that is not distorted on intertemporal allocation. If, in contrast, it is L-region that is not distorted on intertemporal allocation, we find that the recipient region of interregional redistribution should borrow more than the contributor region, and it must face a public debt floor in order to implement the asymmetric-information optimum. There is no such indeterminacy of the top-type region in the second-best context considered by Huber and Runkel, which enables them to obtain a clear-cut result concerning the optimal local borrowing rules. However, under the informational asymmetry induced by IPGs as well as the joint consideration of two opposite intergenerational externalities, either type of region could have incentives to mimic the other type of region, or equivalently either type of region could be the top-type, and hence the spending decision of either type of region would need to be distorted (either downward or upward) to guarantee self-selection in the course of implementation. We thus establish more comprehensive local borrowing rules than those suggested by Huber and Runkel (2008). Therefore, the main novelty of this paper is in accounting for a positive intergenerational externality generated by IPGs and sharpening the results of Huber and Runkel in this more realistic setting.

[6] This is a well-known phrase coined by Zhu Rongji, the former premier of the People's Republic of China, on a visit to Jiujiang City, Jiangxi Province to describe a jerry-built dam.

Moreover, our paper is related to the studies of Rangel (2003, 2005) on how to protect future generations from expropriation and to induce optimal investment of IPGs.[7] Rangel (2003) showed that backward intergenerational goods, such as social security, play a crucial role in sustaining investment in the IPGs, while Rangel (2005) arrived at a similar conclusion by introducing constitutional restrictions on the tax base. As a common feature, these two studies focus on the governance of a single level (either the national level or the regional level), whereas we design institutional arrangements to alleviate the lack of intergenerational incentives from the federalism perspective. We show that appropriate interregional redistribution scheme and regional budget institutions can be designed such that both regional over-borrowing and inefficiently-low provision of local IPGs can be prevented from happening. This could be regarded as another contribution of our paper.

The rest of the paper is organized as follows. Section 2 describes the model. In Sect. 3, we derive the optimal interregional redistribution and regional debt policies under both complete information and asymmetric information. In Sect. 4, we show how these welfare optima can be implemented through decentralized debt decisions and federal redistribution. Section 5 concludes. Proofs are relegated to "Appendix A".

2 Model

We consider a two-period environment of a federation consisting of a federal government (also referred to as the center) and two regions, indexed A and B, respectively. They have the same period-1 population size that is normalized to one for notational simplicity. Each region is populated by a cohort of identical individuals who live for one period only, and there is a new cohort of exactly the same size in period 2. In period $t \in \{1, 2\}$, each individual has a given income $y_t > 0$. For a region $R \in \{A, B\}$, the regional social welfare is given by

$$u_1\left(c_1^R\right) + g_1\left(G_1^R\right) + u_2\left(c_2^R\right) + g_2\left(\theta^R G_1^R + G_2^R\right), \tag{1}$$

in which c_1^R and c_2^R are private consumptions, G_1^R and G_2^R are public goods, and $\theta^R \in (0, 1]$ is a parameter measuring the degree of intergenerational externality of the IPG, G_1^R.[8] All four functions in (1) are strictly increasing and strictly concave, and satisfy the usual Inada conditions. Also, note that the preference specification encompasses

[7] Some other related studies include Schultz and Sjöström (2001), Hatfield (2008, 2014), Conley et al. (2019), among others. They, nevertheless, focus on other mechanisms, rather than the federal transfers and budget institutions considered here, to internalize the intergenerational externality. Importantly, they implicitly assume away the friction brought about by informational asymmetries, which however define the key feature of the current economic environment.

[8] IPG is a kind of public good produced in generation 1 and still (partially) usable in generation 2 (e.g., Rangel 2005). To focus on the primary concern of the paper, we shall not consider the possibility that local IPGs may generate positive spillovers across jurisdictions (see, e.g., Cremer et al. 1997; Bloch and Zenginobuz 2006, 2007). A more involved analysis may take into account both intergenerational and inter-jurisdictional spillovers of local public goods provision, which however is beyond the scope of the present paper and is left for future research.

the special case with $u_2(c_2^R) + g_2(\theta^R G_1^R + G_2^R) \equiv \beta[u_1(c_2^R) + g_1(\theta^R G_1^R + G_2^R)]$, in which $\beta > 0$ is a social discount factor that can be interpreted as a political parameter reflecting the degree to which regional governments take into account the welfare of future generations. In particular, the case with $\beta < 1$ characterizes the fact that present generations are imperfectly altruistic (e.g., Altonji et al. 1992, 1997).[9]

Throughout, we impose the following:

Assumption 2.1 These two regions differ only in θ with $\theta^A < \theta^B$.

This assumption can be interpreted from the following perspective. The IPGs in region B (H-region) are of higher quality than those in region A (L-region), and hence a higher fraction of the IPGs is still usable in period 2.

An individual of generation t in region R has private budget constraint $c_t^R + \tau_t^R = y_t$. The lump sum tax τ_t^R is collected by the local government to finance the provision of local public goods. In period 1, region R receives a transfer z^R from the center and issues debt b^R. Debt plus interest has to be repayed in period 2, taking as given the common interest rate $r > 0$.[10] The fiscal budget constraints of region R in periods 1 and 2 can be written as $G_1^R = \tau_1^R + b^R + z^R$ and $G_2^R = \tau_2^R - (1 + r)b^R$, respectively. If the transfer from the center is negative, then it means that the local government has to pay a tax to the center. Under pure redistribution, the budget constraint of the center is

$$z^A + z^B = 0, \tag{2}$$

which means that the center collects resources from one region to finance transfers to the other region.

Region R's problem is, for given b^R and z^R chosen by the center, choosing taxes τ_1^R and τ_2^R (or equivalently choosing consumption levels c_1^R and c_2^R) to maximize regional welfare (1) subject to the two private budget constraints as well as two regional fiscal budget constraints described above.

In the remaining part of this model, the region index R is suppressed to simplify notation. Combining the private budget constraints with the public budget constraints and applying them to Eq. (1), a region's value function is given by

$$\begin{aligned} V(b, z, \theta) \equiv \max_{c_1, c_2} \; & u_1(c_1) + g_1(y_1 + b + z - c_1) + u_2(c_2) \\ & + g_2(\theta(y_1 + b + z - c_1) + y_2 - b(1 + r) - c_2). \end{aligned} \tag{3}$$

The first-order conditions are thus written as

$$u_1'(c_1) = g_1'(G_1) + \theta g_2'(\theta G_1 + G_2) \quad \text{and} \quad u_2'(c_2) = g_2'(\theta G_1 + G_2), \tag{4}$$

[9] Also, if we assume that voting occurs at the beginning of each period, then it can be interpreted as the current politician in power having a less-than-one probability (exogenously given), β, to be reelected in the future.

[10] It seems reasonable to assume that there is a common capital market within a federation. So, there is a single price level of capital to eliminate arbitrage opportunities.

which also represent the Samuelson condition for the optimal provision of public goods. We can thus write optimal private consumptions as functions of debt, transfers and the degree of intergenerational spillovers: $c_1 \equiv \phi(b, z, \theta)$ and $c_2 \equiv \psi(b, z, \theta)$.

As the final component of the model, we have as shown in the "Appendix A" that

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0}\right) < 0 \quad \text{for all } \theta,$$

which combined with Assumption 2.1 implies that, in every point in the (b, z)-space, the slope of the indifference curve of region A is larger than that of the indifference curve of region B. This also guarantees the single-crossing property required for solving adverse selection problems.

3 Optimal interregional redistribution and local debt policy

Here we characterize the optimal constrained allocation chosen by the federal government, and then show how to implement it using transfers and bounds on the public debt chosen by the regions in the next section.

We assume that the federal government cannot observe each region's degree of intergenerational externality, but it is public knowledge that θ belongs to the set $\{\theta^A, \theta^B\}$ with $\theta^A < \theta^B$ and that they cannot be of the same type. Applying the revelation principle, the center offers each local government a contract stipulating the federal transfer and the region's debt. The timing of game reads as follows:

- These two local governments privately observe θ^R.
- The federal government offers two contracts: $\{b^A, z^A\}$ and $\{b^B, z^B\}$ (or equivalently $\{b(\theta), z(\theta)\}$ for $\theta \in \{\theta^A, \theta^B\}$).
- These two local governments simultaneously pick a contract (or equivalently report their types), and the game ends.

Formally, since we assume that there are just two regions (namely two agents), the center (namely the principal) thus solves the following maximization problem:[11]

$$\max_{b^A, z^A, b^B, z^B} \quad V(b^A, z^A, \theta^A) + V(b^B, z^B, \theta^B)$$

[11] Following the common practice in the literature, such as Lockwood (1999), Huber and Runkel (2008) and Dai et al. (2019a), participation constraints are ignored. In practice, it is politically and/or economically costly for a region to leave the federation. Formally, the participation constraints in the current context can be written as:

$$V(b^A, z^A, \theta^A) \geq \max_{b^A} \tilde{V}(b^A, \theta^A) \quad (\mathrm{IR}_A);$$

$$V(b^B, z^B, \theta^B) \geq \max_{b^B} \tilde{V}(b^B, \theta^B) \quad (\mathrm{IR}_B);$$

in which

subject to the federal fiscal budget constraint (2) and these incentive-compatibility constraints:

$$
\begin{aligned}
V(b^A, z^A, \theta^A) &\geq V(b^B, z^B, \theta^A) \quad (\mathrm{IC}_A);\\
V(b^B, z^B, \theta^B) &\geq V(b^A, z^A, \theta^B) \quad (\mathrm{IC}_B).
\end{aligned}
$$

The Lagrangian can thus be written as

$$
\begin{aligned}
\mathcal{L}(b^A, z^A, b^B, z^B; \mu_A, \mu_B, \lambda) = \;& (1+\mu_A)V(b^A, z^A, \theta^A) - \mu_A V(b^B, z^B, \theta^A)\\
&+ (1+\mu_B)V(b^B, z^B, \theta^B) - \mu_B V(b^A, z^A, \theta^B)\\
&+ \lambda(0 - z^A - z^B),
\end{aligned} \tag{5}
$$

in which μ_A, μ_B and λ are Lagrangian multipliers. Without loss of generality, we let the federal budget constraint (2) be binding so that $\lambda > 0$, which can be interpreted as the shadow price of federal revenues.

As a standard benchmark, we consider first the case with complete information between the center and local governments. To do this, we simply ignore the IC constraints above and let $\mu_A = \mu_B = 0$ in (5). We index the first-best allocation by the superscript ^{FB}.

Proposition 3.1 *Under Assumption 2.1 and complete information, the first-best optimum satisfies:*

(i) *The intertemporal rate of substitution equals intertemporal rate of transformation, namely*

$$
\frac{g_1'(G_1^{R,FB})}{g_2'(\theta^R G_1^{R,FB} + G_2^{R,FB})} = 1 + r - \theta^R \quad \textit{for any} \quad R \in \{A, B\};
$$

(ii) $c_1^{A,FB} = c_1^{B,FB}$, $G_1^{A,FB} < G_1^{B,FB}$, $c_2^{A,FB} = c_2^{B,FB}$, $G_2^{A,FB} > G_2^{B,FB}$, $\theta^A G_1^{A,FB} + G_2^{A,FB} = \theta^B G_1^{B,FB} + G_2^{B,FB}$, *and* $b^{A,FB} < b^{B,FB}$.

Proof See "Appendix A". □

Footnote 11 continued

$$
\begin{aligned}
\tilde{V}\left(b^R, \theta^R\right) \equiv \max_{c_1^R, c_2^R} \; & u_1\left(c_1^R\right) + g_1\left(y_1 + b^R - c_1^R\right) + u_2\left(c_2^R\right)\\
&+ g_2\left(\theta^R(y_1 + b^R - c_1^R) + y_2 - b^R(1+r) - c_2^R\right)
\end{aligned}
$$

for $R \in \{A, B\}$. That is, individual rationality is satisfied only when the regional value under the debt and interregional redistribution policies of the center is no smaller than the highest possible value a region could obtain from leaving the federation. Obviously, under complete information, by setting $z^A = z^B = 0$ in the optimization problem, the center can always replicate what these two regions could get by seceding. As such, the participation constraints can never bind and hence are ignored in establishing the following Proposition 3.1. For the case of asymmetric information, we have identified in "Appendix B" the conditions such that the following Proposition 3.3 holds true even if these participation constraints are binding.

In the first-best optimum, the intertemporal allocation is not distorted in the sense that the intertemproal marginal rate of substitution between first-period public consumption and second-period public consumption equals the intertemporal rate of transformation for both regions. The optimal contract offered by the center features a higher level of debt and more IPG investment allocated to H-region than to L-region, regardless of individual preference structure. Also, H-region turns out to get a strictly higher level of regional welfare than L-region under this optimal contract.

The first-best interregional redistribution policy, however, turns out to depend on the type of preferences for public goods, as stated in the following proposition.[12]

Proposition 3.2 *Under Assumption 2.1 and complete information, the first-best interregional redistribution policy satisfies:*

(i) *Under logarithmic utility function of public goods, namely* $g_1(\cdot) \equiv \ln(\cdot)$ *and* $g_2(\cdot) \equiv \delta \ln(\cdot)$*, we have* $z^{A,FB} = z^{B,FB} = 0$*, and hence the redistribution policy should not be used at all;*

(ii) *Under power utility function of public goods, namely* $g_1(\cdot) \equiv (\cdot)^\alpha$ *and* $g_2(\cdot) \equiv \delta(\cdot)^\alpha$ *with* $\alpha \in (0, 1)$*, we have* $z^{A,FB} < 0 < z^{B,FB}$*, and hence redistribution should be from L-region to H-region;*

(iii) *Under exponential utility function of public goods, namely* $g_1(\cdot) \equiv (-1/\gamma)e^{-\gamma(\cdot)}$ *and* $g_2(\cdot) \equiv -(\delta/\gamma)e^{-\gamma(\cdot)}$ *with* $\gamma > 0$ *and* $\theta^A \geq 1 + r - e^{-1} \cong 0.632 + r$*,*[13] *we have* $z^{A,FB} > 0 > z^{B,FB}$*, and hence redistribution should be from H-region to L-region;*

in which $\delta \in (0, 1]$ *denotes the common discount factor.*

Proof See "Appendix A". □

Proposition 3.2 states that in the full-information benchmark the optimal interregional redistribution policy changes with individual preferences for public goods. Since the benevolent federal government does not exhibit interregional inequality aversion, it seems that alternative preferences have substantially different efficiency implications for the use of interregional redistribution policy. In particular, part (i) shows that the regional heterogeneity in terms of the degree of positive intergenerational spillovers induced by IPGs does not suffice to justify the use of interregional redistribution policy. In other words, under such regional heterogeneity, it is inefficient to enforce any extent of interregional redistribution when individuals have log preference of public goods consumption. In addition, parts (ii)–(iii) show that the direction of efficient interregional redistribution, or the right type of region that pays a positive amount of transfers to the other region, could reverse under alternative preferences. An implication for the following analysis is thus that we might need to refer to preferences when judging which type's IC constraint binds (or which type is the so-called "top type") in the asymmetric-information optimum.

[12] As is well-known, log and power utility functions are CRRA-type preferences, while exponential utility function is a CARA-type preference. Though we shall not consider risks in this context, individual risk preferences do affect the optimal direction of interregional redistribution under complete information. Indeed, the optimal direction of redistribution under complete information tends to reverse from CRRA to CARA utility of public goods.

[13] The last condition is satisfied when $r < 0.368$, and θ^A is sufficiently large.

Under asymmetric information, the parameter measuring the degree of intergenerational externality is private information so that the local government of a given region could mimic the local government of the other region in order to obtain transfers. We now index the second-best allocation by the superscript *.

Proposition 3.3 *Under Assumption 2.1 and asymmetric information, the second-best optimum satisfies:*

(i) *Suppose $\mu_A > 0$, then we have*

- (i-a) *The intertemporal rate of substitution equals the intertemporal rate of transformation in region A, i.e., $g_1'(G_1^{A*}) = (1 + r - \theta^A) g_2'(\theta^A G_1^{A*} + G_2^{A*})$;*
- (i-b) *The intertemporal rate of substitution is smaller than the intertemporal rate of transformation in region B, i.e., $g_1'(G_1^{B*}) < (1 + r - \theta^B) g_2'(\theta^B G_1^{B*} + G_2^{B*})$;*
- (i-c) $c_1^{A*} > c_1^{B*}$, $G_1^{A*} < G_1^{B*}$, $c_2^{A*} > c_2^{B*}$, $G_2^{A*} > G_2^{B*}$, $\theta^A G_1^{A*} + G_2^{A*} > \theta^B G_1^{B*} + G_2^{B*}$, $b^{A*} < b^{B*}$, *and* $z^{A*} < 0 < z^{B*}$.

(ii) *Suppose $\mu_A = 0$ and $\mu_B > 0$, then we have*

- (ii-a) *The intertemporal rate of substitution is greater than the intertemporal rate of transformation in region A, i.e., $g_1'(G_1^{A*}) > (1 + r - \theta^A) g_2'(\theta^A G_1^{A*} + G_2^{A*})$;*
- (ii-b) *The intertemporal rate of substitution equals the intertemporal rate of transformation in region B, i.e., $g_1'(G_1^{B*}) = (1 + r - \theta^B) g_2'(\theta^B G_1^{B*} + G_2^{B*})$;*
- (ii-c) $c_1^{A*} < c_1^{B*}$, $b^{A*} < b^{B*}$ *and* $z^{A*} > 0 > z^{B*}$. *Moreover, if the decreasing part of the indifference curve of region B in the (b, z)-space is steep enough, then $G_1^{B*} < G_1^{A*} \leq [g_2'(\hat{x})/g_2'(\check{x})] G_1^{B*}$ for some $\hat{x} \in (\theta^A G_1^{B*} + G_2^{B*}, \theta^B G_1^{B*} + G_2^{B*})$ and some $\check{x} \in (\theta^A G_1^{A*} + G_2^{A*}, \theta^B G_1^{A*} + G_2^{A*})$, $c_2^{A*} > c_2^{B*}$, and $\theta^A G_1^{A*} + G_2^{A*} > \theta^B G_1^{B*} + G_2^{B*}$. However, if $G_1^{A*} > [g_2'(\hat{x})/g_2'(\check{x})] G_1^{B*}$, then only part (i) holds true.*

Proof See "Appendix A". □

In the second-best optimum, the intertemporal allocation is not distorted for the region whose incentive-compatibility constraint is binding; for the other region, it is distorted such that the marginal rate of substitution between period-1 public consumption and period-2 public consumption is either smaller or larger than the intertemporal rate of transformation, depending on the type of the region under consideration. Without imposing further restrictions on individual preference structure and the scale of regional difference, either one of the two regions could be interpreted as the "top" type, and hence the property of no-distortion-at-the-top[14] applies to it. As is customary in the mechanism design literature, the top-type region will provide a positive amount of transfers to the other region, thereby characterizing the second-best interregional redistribution policy. Moreover, regardless of whether L-region is distorted

[14] See, e.g., Laffont and Martimort (2002).

on intertemporal allocation or not, H-region should borrow more than it borrows.[15] This feature holds true in both the complete-information and asymmetric-information optimum, and hence is robust to the introduction of the current type of informational asymmetry between the center and regions.

In particular, we have shown in the proof of part (i) that if it is L-region that is not distorted on intertemporal allocation, then H-region must be distorted, regardless of the underlying preference structure. That is, if L-region is the top-type region, then the IC constraint of H-region cannot be binding. In part (ii-c), we have derived a sufficient condition, namely $G_1^{A*} > [g_2'(\hat{x})/g_2'(\check{x})]G_1^{B*}$ or equivalently $G_1^{A*}/G_1^{B*} > g_2'(\hat{x})/g_2'(\check{x})$, under which L-region becomes the top-type region. This condition basically requires a lower bound imposed on the IPG investment of L-region. To intuitively understand this claim is equivalent to understanding why L-region has incentives to mimic H-region under this condition. Indeed, if L-region misreports its type by mimicking H-region, a high IPG investment implies the debt issued is larger than the transfers paid in generation 1, and also that the downward distortion placed on generation 2 is bounded because of large positive intergenerational spillovers generated by the IPGs provided by generation 1. As such, L-region may obtain a higher level of regional welfare from cheating than from telling the truth.

In addition, we see that the smaller $g_2'(\hat{x})/g_2'(\check{x})$ is, the sufficient condition given above is more likely to be fulfilled. Under those special preferences of public goods given in Proposition 3.2, we have $g_2'(\hat{x})/g_2'(\check{x}) = \check{x}/\hat{x} > 1$ under log utility, $g_2'(\hat{x})/g_2'(\check{x}) = (\check{x}/\hat{x})^{1-\alpha}$ under power utility, and $g_2'(\hat{x})/g_2'(\check{x}) = \exp[\gamma(\check{x} - \hat{x})]$ under exponential utility. As is obvious, the sufficient condition is most likely to be fulfilled under power utility, is least likely to be fulfilled under exponential utility, and is intermediate likely to be fulfilled under log utility. Therefore, these findings are somewhat consistent with what we obtained in Proposition 3.2 under complete information.

4 Implementation of welfare optimum

We have established the welfare optimum under both complete and asymmetric information, we now analyze how to implement it via regionally-decentralized debt

[15] In fact, this feature also holds in the case of a continuum of types such that the corresponding second-order sufficient condition for incentive compatibility is satisfied. After some algebra, the local second-order condition can be written as

$$\dot{b}(\theta) \cdot V_z(b(\theta), z(\theta), \theta) \cdot \frac{\partial}{\partial \tilde{\theta}} \left(\frac{V_b(b(\theta), z(\theta), \tilde{\theta})}{V_z(b(\theta), z(\theta), \tilde{\theta})} \right) \bigg|_{\tilde{\theta}=\theta} \geq 0.$$

Noting that $V_z(\cdot) = g_1' + \theta g_2' > 0$ and the Spence-Mirrlees property reads as

$$\frac{\partial}{\partial \theta} \left(\frac{V_b}{V_z} \right) = \frac{(1+r)[(g_2')^2 - G_1 g_1' g_2'']}{(g_1' + \theta g_2')^2} > 0,$$

we thus must have $\dot{b}(\theta) \geq 0$, as desired. It is easy to verify that the local second-order condition also implies global optimality of the truth-telling strategy with the help of the above Spence-Mirrlees property.

decisions. That is, the optimality problem in the previous section has the center choose both the local debt and the interregional transfers, while here we let local governments choose a level of public debt to maximize their regional welfare, taking as given the redistribution scheme enforced by the federal government. Formally, the maximization problem of region $R = A$ or B is

$$\max_{b^R} \; V(b^R, z, \theta^R)$$

for any given z. The first-order condition is thus written as

$$\begin{aligned} g_1'\left(y_1 + b^R + z - \phi(b^R, z, \theta^R)\right) = &\left(1 + r - \theta^R\right) \\ &\times g_2'\Big(\theta^R(y_1 + b^R + z - \phi(b^R, z, \theta^R)) \\ &+ y_2 - (1+r)b^R - \psi(b^R, z, \theta^R)\Big), \end{aligned} \tag{6}$$

showing that the intertemporal rate of substitution must be equal to the intertemporal rate of transformation at the regional welfare optimum.

Making use of (6) and Proposition 3.1, we immediately have the following: The complete-information optimum is attained by simply setting $z^A = z^{A,FB}$ and $z^B = z^{B,FB}$. The reason is that the center can observe the type of each region and also the complete-information optimum does not distort the intertemporal allocation desired by each region.

Under asymmetric information, the center should design redistribution scheme that guarantees incentive compatibility for both regions. It follows from Proposition 3.3 that the intertemporal allocation of some region is distorted in the asymmetric-information optimum, so the asymmetric-information optimum can no longer be implemented by decentralized debt decisions characterized by (6) with the center simply setting $z^A = z^{A*}$ and $z^B = z^{B*}$. Indeed, certain institutional restriction must be imposed on regional borrowing decisions.

Proposition 4.1 *Under Assumption 2.1, the following statements are true.*

(i) *Suppose $\mu_A > 0$, then the asymmetric-information optimum is attained by setting $z^A = z^{A*}$, $z^B = z^{B*}$ and a lower bound $\underline{b} \equiv b^{B*}$ on the public debt of region B.*
(ii) *Suppose $\mu_A = 0$ and $\mu_B > 0$, then the asymmetric-information optimum is attained by setting $z^A = z^{A*}$, $z^B = z^{B*}$ and an upper bound $\overline{b} \equiv b^{A*}$ on the public debt of region A.*

Proof See "Appendix A". □

Note from Proposition 3.3 that $b^{A*} < b^{B*}$, so the public debt floor $\underline{b} \equiv b^{B*}$ under $\mu_A > 0$ applies to H-region only, while the public debt ceiling $\overline{b} \equiv b^{A*}$ under $\mu_A = 0$ and $\mu_B > 0$ applies to L-region only. The fiscal constraint of a debt floor distorts the spending decision of H-region in favor of future public consumption, and hence the investment in IPGs, which makes explicit the implicit borrowing constraint contained in the asymmetric-information optimum. Also, such level of debt floor renders the allocation of H-region unattractive for L-region, who actually faces a higher opportunity cost of borrowing, so that it voluntarily pays the lump-sum tax to the center instead of mimicking H-region. The fiscal constraint of a debt ceiling that distorts the spending decision of L-region in favor of current public consumption can be intuitively understood in a similar way. In a word, imposing these regional budget constraints helps to resolve the self-selection problem of the center in the presence of the current type of asymmetric information.

In addition, it follows from part (i) of Proposition 3.3 that it is the recipient region other than the contributor region of interregional redistribution that faces this debt floor. Similarly, it follows from part (ii) of Proposition 3.3 that it is the recipient region other than the contributor region of interregional redistribution that faces this debt ceiling. Interestingly, finding (i) seems to be in stark contrast to while finding (ii) seems to be consistent with that obtained by Huber and Runkel (2008) in a similar context but with regional private information on social discount factor and without positive intergenerational externalities generated by IPGs. In their setting, truth-telling implementation calls for a debt ceiling placed on the recipient region of redistribution.

5 Concluding summary

This paper develops a two-period theoretical model of a federation consisting of a federal government and two regions that differ in the degree of intergenerational externality induced by their own IPG investment. The amount of IPG investment is public knowledge whereas the degree of the resulting intergenerational spillovers is only observable to each region. From the endowment/technology perspective, the degree of intergenerational spillovers measures the quality (and hence sustainable usability) of IPGs. So, a higher degree of spillovers implies a lower depreciation rate. Importantly, it is reasonable to interpret the quality of local IPGs as the private information of each region.

We first establish the complete-information and the asymmetric-information welfare optimum, and then propose budget arrangements so that the federal government can (truthfully) implement the asymmetric-information welfare optimum through decentralized regional debt decisions and federal transfers. The main results are summarized as follows. First, H-region should issue more public debt in the first generation than does L-region, regardless of whether their degrees of intergenerational spillovers are observable by the center. Second, in the asymmetric-information optimum, the region that is not distorted on intertemporal allocation should be the contributor of interregional redistribution, regardless of the type of the region under consideration. In particular, if IPG investment in L-region relative to that in H-region is above some threshold, then only L-region is not distorted on intertemporal allocation and extracts the information rent. And third, to implement the asymmetric-information optimum with regions having autonomy in the choice of their public debt, differentiated budget institutions must be imposed: If H-region is distorted on intertemporal allocation, then it faces a debt floor; if, however, L-region is distorted, then it faces a debt ceiling.

In a similar context in which the local public goods are non-durable and the regional discount rate (instead of the durability of the public goods) is local governments' private information, Huber and Runkel (2008) established differentiated budget constraints that place strict debt limits for recipients and lax debt limits for contributors of interregional redistribution. We extend the insightful analysis of Huber and Runkel by introducing durable local public goods (i.e., intergenerational public goods or IPGs) and analyzing the implications for the federal government's interregional redistribution policy and regional debt policies from regional differences in the durability of the IPGs. This situation is interesting and unique in that the positive effect of the IPG investment on the future generation can offset the negative effect of government borrowing on the future generation. In this context, we obtain a novel result about optimal differentiated budget constraints, under reasonable conditions, that place a strict debt floor (rather than a debt limit or debt ceiling) for the recipient region of the federal transfers. The quantitative importance of our argument is of course an empirical question which, however, goes beyond the scope of our paper and is left for future research.[16]

We admit the practical relevance of those arguments in favor of debt limits, as suggested by Rogoff (1990), Tabellini and Alesina (1990), Chari and Kehoe (2007), and among others, here we propose a doubt that such institutional arrangements are incomplete in the sense that they are likely to discourage local governments and result in inefficiently low provision of IPGs.[17] In a different setting with emphasizing a moral hazard problem, rather than the current adverse selection problem, between the center and regional governments, Besfamille and Lockwood (2008) show that hard budget constraints might generate inefficiencies leading to underprovision of local projects. As such, when considered more comprehensively, a hard budget constraint on local governments is not always best. Roughly speaking, the current argument partially justifies why the economists need to reconsider how much governments can borrow (see, The Economist 2019).[18]

Appendix A: Proofs

Proof of $\mathrm{d}^2 z/\mathrm{d}b^2|_{dV=0} > 0$ We establish first *the strict convexity of the indifference curves* in the (b, z)-space for both types of regions. As shall be clear soon, this property is of crucial importance for proving the formal results in Sect. 3.

[16] There are some studies that empirically investigated the effects of differentiated budget institutions, such as Poterba (1994) and Poterba and Rueben (2001), but they did not identify (or take into account) the effects on the provision of local IPGs.

[17] For example, the empirical evidence from China has somehow illustrated this effect (Li 2018).

[18] Indeed, these debt limits are often violated in practice. In the European Union, for instance, Germany and France violated the 3% (debt-to-GDP ratio agreed-upon) criterion in 2003 (see, Huber and Runkel 2008). Also, it is well recognized that local governments in China tend to be over-borrowing. Due to political career concerns, local politicians in China have incentives to issue high levels of public debt and invest in productive public infrastructure to stimulate (short-term) GDP growth so that they have a higher probability to be promoted to higher levels of the bureaucratic hierarchy.

Applying the Implicit Function Theorem to (4), we have these partial derivatives:

$$\begin{aligned}\phi_b(b,z,\theta) &= \frac{g_1''(u_2''+g_2'')-(1+r-\theta)\theta u_2'' g_2''}{\Sigma},\\ \psi_b(b,z,\theta) &= \frac{\theta u_1'' g_2''-(1+r)(u_1''+g_1'')g_2''}{\Sigma};\end{aligned} \tag{7}$$

and

$$\phi_z(b,z,\theta)=\frac{g_1''(u_2''+g_2'')+\theta^2 u_2'' g_2''}{\Sigma},\quad \psi_z(b,z,\theta)=\frac{\theta u_1'' g_2''}{\Sigma}; \tag{8}$$

with $\Sigma \equiv (u_1''+g_1'')(u_2''+g_2'')+\theta^2 u_2'' g_2'' > 0$. Applying the Envelope Theorem to (3), we obtain the slope of an indifference curve in the (b,z)-space as

$$\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0} = -\frac{g_1'\left(\hat{G}_1\right)-(1+r-\theta)g_2'\left(\theta\hat{G}_1+\hat{G}_2\right)}{g_1'\left(\hat{G}_1\right)+\theta g_2'\left(\theta\hat{G}_1+\hat{G}_2\right)} \tag{9}$$

for $\hat{G}_1 \equiv y_1+b+z-\phi(b,z,\theta)$ and $\hat{G}_2 \equiv y_2-b(1+r)-\psi(b,z,\theta)$. By (9), we then have

$$\begin{aligned}\left.\frac{\mathrm{d}^2z}{\mathrm{d}b^2}\right|_{dV=0} =\ & \frac{(1+r)g_2''}{g_1'+\theta g_2'}\\ &\times\left[\theta\left(1+\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0}-\phi_b-\phi_z\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0}\right)-1-r-\psi_b-\psi_z\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0}\right]\\ &-\frac{(1+r)g_2'g_1''}{(g_1'+\theta g_2')^2}\left(1+\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0}-\phi_b-\phi_z\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0}\right)-\frac{\theta(1+r)g_2'g_2''}{(g_1'+\theta g_2')^2}\\ &\times\left[\theta\left(1+\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0}-\phi_b-\phi_z\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0}\right)-1-r-\psi_b-\psi_z\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0}\right],\end{aligned} \tag{10}$$

in which ϕ_b, ϕ_z, ψ_b and ψ_z represent the partial derivatives of individual private consumptions with respect to b and z, respectively. Using (9), (7) and (8) yields the following:

$$1+\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0}-\phi_b-\phi_z\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0}=\frac{(1+r)\theta u_2'' g_2''}{\Sigma}+\frac{(1+r)u_1''(u_2''+g_2'')g_2'}{\Sigma(g_1'+\theta g_2')}$$

and

$$1+r+\psi_b+\psi_z\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0}=\frac{(1+r)(u_1''+g_1''+\theta^2 g_2'')u_2''}{\Sigma}+\frac{\theta(1+r)u_1''g_2''g_2'}{\Sigma(g_1'+\theta g_2')}.$$

Plugging these terms in (10) and simplifying the algebra, we have

$$\left.\frac{\mathrm{d}^2z}{\mathrm{d}b^2}\right|_{dV=0}=-(1+r)^2\frac{Q}{\Sigma(g_1'+\theta g_2')^3}>0$$

for

$$Q \equiv g_1' g_2'' u_2'' (u_1'' g_1' + g_1'' g_1' + \theta g_1'' g_2') + \theta g_2' u_2'' g_1'' g_2'' (g_1' + \theta g_2')$$
$$+ u_1'' u_2'' g_1'' (g_1')^2 + (u_1'')^2 u_2'' g_1'' g_2'' (g_2')^2 < 0.$$

So the indifference curve is U-shaped with the minimum at the point where the intertemporal rate of substitution equals the intertemporal rate of transformation, i.e., $g_1'(G_1)/g_2'(\theta G_1 + G_2) = 1 + r - \theta$.

Proof of $\mathrm{d}(\mathrm{d}z/\mathrm{d}b|_{dV=0})/\mathrm{d}\theta < 0$ for $\forall\theta$. Differentiating (9) with respect to θ and simplifying the algebra, we get

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\left.\frac{\mathrm{d}z}{\mathrm{d}b}\right|_{dV=0}\right) = (1+r)\frac{g_1' g_2'' (G_1 - \theta\phi_\theta - \psi_\theta) + g_2'(g_1''\phi_\theta - g_2')}{(g_1' + \theta g_2')^2}, \tag{11}$$

in which ϕ_θ and ψ_θ represent the partial derivatives of individual private consumptions with respect to θ. Applying Implicit Function Theorem to (4) produces

$$\phi_\theta = \frac{1}{\Sigma}[g_2'(u_2'' + g_2'') + \theta G_1 u_2'' g_2''] \quad \text{and} \quad \psi_\theta = \frac{1}{\Sigma}[G_1 g_2''(u_1'' + g_1'') - \theta g_2' g_2''],$$

in which $\Sigma > 0$ is given in the previous proof. We then have:

$$G_1 - \theta\phi_\theta - \psi_\theta \;=\; \frac{G_1 u_2''(u_1'' + g_1'') - \theta g_2' u_2''}{\Sigma} \;>\; 0$$

and

$$g_1''\phi_\theta - g_2' \;=\; \frac{\theta G_1 u_2'' g_1'' g_2'' - g_2' u_1''(u_2'' + g_2'') - \theta^2 g_2' g_2'' u_2''}{\Sigma} \;<\; 0.$$

Substituting these two terms into Eq. (11) leads to the desired assertion.

Proof of Proposition 3.1 Using (3) and (5) gives the following FOCs with respect to b^A and z^A, respectively:

$$\begin{aligned}&(1+\mu_A)[g_1'(G_1^A) - (1+r-\theta^A)g_2'(\theta^A G_1^A + G_2^A)]\\ &\quad = \mu_B[g_1'(G_1^A) - (1+r-\theta^B)g_2'(\theta^B G_1^A + G_2^A)];\ \text{and}\\ &\quad (1+\mu_A)[g_1'(G_1^A) + \theta^A g_2'(\theta^A G_1^A + G_2^A)]\\ &\quad = \mu_B[g_1'(G_1^A) + \theta^B g_2'(\theta^B G_1^A + G_2^A)] + \lambda.\end{aligned} \tag{12}$$

Those of region B can be obtained by symmetry:

$$\begin{aligned}&(1+\mu_B)[g_1'(G_1^B) - (1+r-\theta^B)g_2'(\theta^B G_1^B + G_2^B)]\\ &\quad = \mu_A[g_1'(G_1^B) - (1+r-\theta^A)g_2'(\theta^A G_1^B + G_2^B)];\ \text{and}\\ &\quad (1+\mu_B)[g_1'(G_1^B) + \theta^B g_2'(\theta^B G_1^B + G_2^B)]\end{aligned}$$

$$= \mu_A[g_1'(G_1^B) + \theta^A g_2'(\theta^A G_1^B + G_2^B)] + \lambda. \tag{13}$$

Imposing $\mu_A = \mu_B = 0$ in (12) and (13), we have

$$\begin{aligned} g_1'(G_1^R) - (1 + r - \theta^R) g_2'(\theta^R G_1^R + G_2^R) = 0, \\ g_1'(G_1^R) + \theta^R g_2'(\theta^R G_1^R + G_2^R) = \lambda, \end{aligned} \tag{14}$$

for any $R \in \{A, B\}$. Part (i) immediately follows from the first equation of (14). It follows that $(1+r)g_2'(\theta^R G_1^R + G_2^R) = \lambda$ for any $R \in \{A, B\}$, and hence $\theta^A G_1^A + G_2^A = \theta^B G_1^B + G_2^B$. This combined with (14) yields that

$$\frac{g_1'(G_1^A)}{g_1'(G_1^B)} = \frac{1 + r - \theta^A}{1 + r - \theta^B} > 1$$

under Assumption 2.1. We thus have $G_1^A < G_1^B$, which combined with Assumption 2.1 produces $G_2^A - G_2^B = \theta^B G_1^B - \theta^A G_1^A > \theta^B (G_1^B - G_1^A) > 0$, as desired. Also, using (4) and (14) immediately gives $u_1'(c_1^R) = \lambda$ and $u_2'(c_2^R) = g_2'(\theta^R G_1^R + G_2^R)$, so $c_1^A = c_1^B$ and $c_2^A = c_2^B$. We then have $0 < G_2^A - G_2^B = (1 + r)(b^B - b^A)$, as desired in part (ii). □

Proof of Proposition 3.2 We shall complete the proof in 3 steps.

Step 1. Under log utility, we get by (14) that $G_2^R = [\delta(1 + r - \theta^R) - \theta^R]G_1^R$ and $G_1^R = (1 + r)/[\lambda(1 + r - \theta^R)]$ for $\forall R$, by which it is easy to verify that $z^A - z^B = G_1^A - G_1^B + b^B - b^A = G_1^A - G_1^B + [(G_2^A - G_2^B)/(1 + r)] = 0$. The proof of part (i) is thus completed.

Step 2. Under the power utility function form, we get by (14) that

$$G_1^R = \left[\frac{\alpha(1 + r)}{\lambda(1 + r - \theta^R)}\right]^{1/(1-\alpha)} \quad \text{and} \quad G_2^R = \left[\frac{\delta\alpha(1 + r)}{\lambda}\right]^{1/(1-\alpha)} - \theta^R G_1^R$$

for $\forall R$. We then have under Assumption 2.1 that

$$\begin{aligned} z^A - z^B = &\left(\frac{1}{1 + r}\right)\left[\frac{\alpha(1 + r)}{\lambda}\right]^{1/(1-\alpha)} \\ &\times \left[\left(\frac{1}{1 + r - \theta^A}\right)^{\alpha/(1-\alpha)} - \left(\frac{1}{1 + r - \theta^B}\right)^{\alpha/(1-\alpha)}\right] < 0, \end{aligned}$$

which combines with (2) produces the desired result in part (ii).

Step 3. Applying the exponential utility function to Eq. (14) gives rise to:

$$G_1^R = -\frac{1}{\gamma}\ln[\delta(1 + r - \theta^R)] - \frac{1}{\gamma}\ln\left[\frac{\lambda}{\delta(1 + r)}\right]$$

and

$$G_2^R = \left(\frac{1-\theta^R}{\gamma}\right)\ln\left[\frac{\delta(1+r)}{\lambda}\right] + \frac{\theta^R}{\gamma}\ln[\delta(1+r-\theta^R)]$$

for any $R = A$ or B. We thus get by simplifying the algebra that

$$\gamma(1+r)(z^A - z^B) = (\theta^B - \theta^A)\ln\left(\frac{1+r}{\lambda}\right) + (1+r-\theta^B)\ln(1+r-\theta^B) \\ - (1+r-\theta^A)\ln(1+r-\theta^A).$$

Applying the exponential utility function to $g_2'(\theta^R G_1^R + G_2^R) = \lambda/(1+r)$ yields $\lambda/[\delta(1+r)] = e^{-\gamma(\theta^R G_1^R + G_2^R)} < 1$, then we have $\lambda < 1+r$. As such, as long as $(1+r-\theta^R)\ln(1+r-\theta^R)$ is nondecreasing in θ^R, then we have $z^A > z^B$ under Assumption 2.1. Noting that $\partial[(1+r-\theta^R)\ln(1+r-\theta^R)]/\partial\theta^R = -\ln[e(1+r-\theta^R)]$, thus $\partial[(1+r-\theta^R)\ln(1+r-\theta^R)]/\partial\theta^R \geq 0$ for $\theta^R \geq 1+r-e^{-1}$, as desired in part (iii). □

Proof of Proposition 3.3 We shall complete the proof in 5 steps.

Step 1 We first prove parts (i-a) and (i-b). Suppose $\mu_A > 0$, namely that region A has incentives to mimic region B and hence IC_A is binding, then as shall be shown in step 3 that we must have IC_B to be not binding so that $\mu_B = 0$. Applying these conditions to (12) and (13) shows that

$$\begin{aligned} g_1'(G_1^A) - (1+r-\theta^A)g_2'(\theta^A G_1^A + G_2^A) = 0, \\ g_1'(G_1^A) + \theta^A g_2'(\theta^A G_1^A + G_2^A) = \frac{\lambda}{1+\mu_A}; \end{aligned} \tag{15}$$

and also

$$\begin{aligned} g_1'(G_1^B) - (1+r-\theta^B)g_2'(\theta^B G_1^B + G_2^B) &= \mu_A[g_1'(G_1^B) \\ &\quad -(1+r-\theta^A)g_2'(\theta^A G_1^B + G_2^B)] \text{ and} \\ &\quad g_1'(G_1^B) + \theta^B g_2'(\theta^B G_1^B + G_2^B) \\ &= \mu_A[g_1'(G_1^B) + \theta^A g_2'(\theta^A G_1^B + G_2^B)] + \lambda. \end{aligned} \tag{16}$$

Then part (i-a) is immediate from (15). Rearranging the equations in (16) gives $0 < \lambda = (1+r)[g_2'(\theta^B G_1^B + G_2^B) - \mu_A g_2'(\theta^A G_1^B + G_2^B)]$, by which we obtain $\mu_A < 1$ under Assumption 2.1. Noting that $-(1+r-\theta)g_2'(\theta G_1 + G_2)$ is strictly increasing in θ, we then have

$$\begin{aligned} &g_1'(G_1^B) - (1+r-\theta^B)g_2'(\theta^B G_1^B + G_2^B) \\ &\quad < \mu_A[g_1'(G_1^B) - (1+r-\theta^B)g_2'(\theta^B G_1^B + G_2^B)] \end{aligned}$$

by using the first equation of (16), $\mu_A > 0$ and Assumption 2.1. In consequence, we must have $(1-\mu_A)[g_1'(G_1^B) - (1+r-\theta^B)g_2'(\theta^B G_1^B + G_2^B)] < 0$, which combined with $\mu_A < 1$ immediately produces part (i-b).

Step 2 We now prove part (i-c). Note that it is assumed that IC_A is binding, then we have that (b^{A*}, z^{A*}) and (b^{B*}, z^{B*}) lie on the same indifference curve of region A in the (b, z)-space. Applying part (i-a) to (9) reveals that (b^{A*}, z^{A*}) lies at the minimum point of this indifference curve. Applying part (i-b) to (9) implies that (b^{B*}, z^{B*}) lies on the increasing part of this indifference curve, so that $b^{B*} > b^{A*}$ and $z^{A*} < 0 < z^{B*}$ given the established strictly-convex property of the indifference curve, as desired.

Rearranging the equations in (15), we can have $g_2'(\theta^A G_1^A + G_2^A) = \lambda/[(1+\mu_A)(1+r)]$. Also, we have shown in step 1 that $\lambda/(1+r) = g_2'(\theta^B G_1^B + G_2^B) - \mu_A g_2'(\theta^A G_1^B + G_2^B)$, by which we have $g_2'(\theta^B G_1^B + G_2^B) > \lambda/[(1+r)(1-\mu_A)]$. As a result, we have $g_2'(\theta^B G_1^B + G_2^B) > g_2'(\theta^A G_1^A + G_2^B)$, which implies $\theta^B G_1^B + G_2^B < \theta^A G_1^A + G_2^A$. This combined with (4) yields that $c_2^{B*} < c_2^{A*}$. Also, noting that $\theta^B G_1^B + G_2^B < \theta^A G_1^A + G_2^A < \theta^B G_1^A + G_2^A$ under Assumption 2.1, then $G_2^A > G_2^B$ when $G_1^B > G_1^A$. Using (15) and (16) again, we have

$$
\begin{aligned}
g_1'(G_1^B) + \theta^B g_2'(\theta^B G_1^B + G_2^B) &= \mu_A[g_1'(G_1^B) + \theta^A g_2'(\theta^A G_1^B + G_2^B)] + \lambda \\
&> \frac{\lambda}{1+\mu_A} = g_1'(G_1^A) + \theta^A g_2'(\theta^A G_1^A + G_2^A),
\end{aligned}
$$

which combined with (4) shows that $u_1'(c_1^B) > u_1'(c_1^A)$, and hence all desired assertions in part (i-c) follow.

Step 3 We now prove that IC_B is not binding, i.e., $V(b^{B*}, z^{B*}, \theta^B) > V(b^{A*}, z^{A*}, \theta^B)$. We shall prove this by means of contradiction. Suppose, in contrast, that $V(b^{B*}, z^{B*}, \theta^B) \le V(b^{A*}, z^{A*}, \theta^B)$, then we have:

$$
\begin{aligned}
g_2(\theta^B G_1^{B*} + G_2^{B*}) - g_2(\theta^B G_1^{A*} + G_2^{A*}) &\le u_1(c_1^{A*}) - u_1(c_1^{B*}) + u_2(c_2^{A*}) - u_2(c_2^{B*}) \\
&\quad + g_1(G_1^{A*}) - g_1(G_1^{B*}) \\
&= g_2(\theta^A G_1^{B*} + G_2^{B*}) - g_2(\theta^A G_1^{A*} + G_2^{A*}), \qquad (17)
\end{aligned}
$$

in which the equality follows from the preassumption that IC_A is binding. Then we get by applying Mean Value Theorem and Assumption 2.1 to (17) that

$$
g_2'(x')G_1^{B*} \ \le\ g_2'(x'')G_1^{A*} \tag{18}
$$

for some $x' \in (\theta^A G_1^{B*} + G_2^{B*}, \theta^B G_1^{B*} + G_2^{B*})$ and some $x'' \in (\theta^A G_1^{A*} + G_2^{A*}, \theta^B G_1^{A*} + G_2^{A*})$. As is obvious, inequality (18) contradicts with $x' < x''$ and $G_1^{B*} > G_1^{A*}$.

Step 4 We now prove parts (ii-a) and (ii-b). Applying $\mu_A = 0$ and $\mu_B > 0$ to (12) and (13) reveals that

$$
\begin{aligned}
g_1'(G_1^B) - (1+r-\theta^B)g_2'(\theta^B G_1^B + G_2^B) &= 0, \\
g_1'(G_1^B) + \theta^B g_2'(\theta^B G_1^B + G_2^B) &= \frac{\lambda}{1+\mu_B};
\end{aligned} \tag{19}
$$

and also

$$g_1'(G_1^A) - (1+r-\theta^A)g_2'(\theta^A G_1^A + G_2^A) = \mu_B[g_1'(G_1^A) - (1+r-\theta^B)g_2'(\theta^B G_1^A + G_2^A)] \text{ and}$$
$$g_1'(G_1^A) + \theta^A g_2'(\theta^A G_1^A + G_2^A)$$
$$= \mu_B[g_1'(G_1^A) + \theta^B g_2'(\theta^B G_1^A + G_2^A)] + \lambda. \quad (20)$$

Part (ii-b) is immediate by using (19). Noting that $-(1+r-\theta^B)g_2'(\theta^B G_1^A + G_2^A) > -(1+r-\theta^A)g_2'(\theta^A G_1^A + G_2^A)$, we then get from (20) that $(1-\mu_B)[g_1'(G_1^A) - (1+r-\theta^A)g_2'(\theta^A G_1^A + G_2^A)] > 0$. Also, using (20) again shows that

$$\mu_B = \frac{g_1'(G_1^A) - (1+r-\theta^A)g_2'(\theta^A G_1^A + G_2^A)}{g_1'(G_1^A) - (1+r-\theta^B)g_2'(\theta^B G_1^A + G_2^A)} < 1.$$

We thus must have $g_1'(G_1^A) > (1+r-\theta^A)g_2'(\theta^A G_1^A + G_2^A)$, as desired in part (ii-a).

Step 5 We now prove parts (ii-c). Applying the result in part (ii-a) to (9) shows that (b^{A*}, z^{A*}) lies on the decreasing part of region B's indifference curve in the (b, z)-space. Since $\mu_B > 0$, and hence IC_B is binding, so using (9) and (19) reveals that (b^{B*}, z^{B*}) lies at the minimum point of region B's indifference curve and also (b^{A*}, z^{A*}) and (b^{B*}, z^{B*}) lie on the same indifference curve. In consequence, the desired assertion on redistribution and debt policy in part (ii-c) follows from the strict convexity of the indifference curve.

Using Eqs. (19) and (20) again, it is easy to see that $g_1'(G_1^A)+\theta^A g_2'(\theta^A G_1^A+G_2^A) > g_1'(G_1^B) + \theta^B g_2'(\theta^B G_1^B + G_2^B)$, which combined with (4) produces that $c_1^{A*} < c_1^{B*}$, as desired. We thus have:

$$G_1^{A*} - G_1^{B*} = \underbrace{b^{A*} - b^{B*}}_{<0} + \underbrace{z^{A*} - z^{B*}}_{>0} + \underbrace{c_1^{B*} - c_1^{A*}}_{>0}.$$

If the decreasing part of the indifference curve of region B in the (b, z)-space is steep enough, then we must have $b^{B*} - b^{A*} \leq z^{A*} - z^{B*}$, and hence $G_1^{A*} > G_1^{B*}$. In fact, the required steepness depends on the curvature of individual utility functions. Using this result and parts (ii-a) and (ii-b), we now have $(1+r-\theta^B)g_2'(\theta^B G_1^{B*} + G_2^{B*}) = g_1'(G_1^{B*}) > g_1'(G_1^{A*}) > (1+r-\theta^A)g_2'(\theta^A G_1^{A*} + G_2^{A*})$, which implies that $\theta^B G_1^{B*} + G_2^{B*} < \theta^A G_1^{A*} + G_2^{A*}$ under Assumption 2.1. So, $c_2^{B*} < c_2^{A*}$ follows from using Eq. (4).

We then proceed to check the incentive-compatibility constraints. Since it is assumed that $\mu_B > 0$, then the complementary slackness condition implies that IC_B is binding, formally

$$g_2(\theta^B G_1^{A*} + G_2^{A*}) - g_2(\theta^B G_1^{B*} + G_2^{B*}) = u_1(c_1^{B*}) - u_1(c_1^{A*}) + u_2(c_2^{B*}) - u_2(c_2^{A*})$$
$$+ g_1(G_1^{B*}) - g_1(G_1^{A*}). \quad (21)$$

Using (21) and IC_A, we thus obtain by applying Mean Value Theorem that

$$V(b^{A*}, z^{A*}, \theta^A) - V(b^{B*}, z^{B*}, \theta^A) = g_2(\theta^B G_1^{B*} + G_2^{B*}) - g_2(\theta^A G_1^{B*} + G_2^{B*})$$

$$
\begin{aligned}
&- [g_2(\theta^B G_1^{A*} + G_2^{A*}) - g_2(\theta^A G_1^{A*} + G_2^{A*})] \\
&= g_2'(\hat{x}) G_1^{B*} - g_2'(\check{x}) G_1^{A*} \geq 0,
\end{aligned}
$$

for some $\hat{x} \in (\theta^A G_1^{B*} + G_2^{B*}, \theta^B G_1^{B*} + G_2^{B*})$ and some $\check{x} \in (\theta^A G_1^{A*} + G_2^{A*}, \theta^B G_1^{A*} + G_2^{A*})$. We, therefore, have $G_1^{A*} \leq [g_2'(\hat{x})/g_2'(\check{x})] G_1^{B*}$ for $g_2'(\hat{x})/g_2'(\check{x}) > 1$. □

Proof of Proposition 4.1 We shall complete the proof in 3 steps.

Step 1 Suppose that region A receives transfer z^{B*} from the center, so its maximization problem is $\max_{b^A} V(b^A, z^{B*}, \theta^A)$ subject to $b^A \geq \underline{b} = b^{B*}$. Applying Envelope Theorem to the region's welfare function, we obtain:

$$
\begin{aligned}
V_b(b^A, z^{B*}, \theta^A) = g_1'(y_1 + b^A + z^{B*} - \phi(b^A, z^{B*}, \theta^A)) - (1 + r - \theta^A) \\
\times g_2'(\theta^A (y_1 + b^A + z^{B*} - \phi(b^A, z^{B*}, \theta^A)) + y_2 \\
-(1 + r) b^A - \psi(b^A, z^{B*}, \theta^A)),
\end{aligned} \tag{22}
$$

and

$$
\begin{aligned}
V_{bb}(b^A, z^{B*}, \theta^A) = g_1'' \cdot [1 - \phi_b(b^A, z^{B*}, \theta^A)] \\
-(1 + r - \theta^A) g_2'' \cdot \{\theta^A [1 - \phi_b(b^A, z^{B*}, \theta^A)] \\
-(1 + r) - \psi_b(b^A, z^{B*}, \theta^A)\}.
\end{aligned} \tag{23}
$$

Noting from applying the Implicit Function Theorem to (4) that

$$
1 - \phi_b(b^A, z^{B*}, \theta^A) = \frac{u_1''(u_2'' + g_2'') + (1 + r)\theta^A u_2'' g_2''}{(u_1'' + g_1'')(u_2'' + g_2'') + (\theta^A)^2 u_2'' g_2''} > 0
$$

and

$$
\begin{aligned}
&1 + r + \psi_b(b^A, z^{B*}, \theta^A) \\
&\quad = \frac{(1 + r)(u_1'' + g_1'') u_2'' + \theta^A g_2'' [u_1'' + (1 + r)\theta^A u_2'']}{(u_1'' + g_1'')(u_2'' + g_2'') + (\theta^A)^2 u_2'' g_2''},
\end{aligned}
$$

we thus have by simplifying the algebra that

$$
\begin{aligned}
&\theta^A [1 - \phi_b(b^A, z^{B*}, \theta^A)] - (1 + r) - \psi_b(b^A, z^{B*}, \theta^A) \\
&\quad = -\frac{[(1 + r - \theta^A) u_1'' + (1 + r) g_1''] u_2''}{(u_1'' + g_1'')(u_2'' + g_2'') + (\theta^A)^2 u_2'' g_2''} < 0.
\end{aligned}
$$

In consequence, $V_{bb}(b^A, z^{B*}, \theta^A) < 0$ for any feasible b^A. Evaluating Eq. (22) at $b^A = b^{B*}$ yields

$$
\begin{aligned}
V_b(b^{B*}, z^{B*}, \theta^A) &= g_1'(G_1^{B*}) - (1 + r - \theta^A) g_2'(\theta^A G_1^{B*} + G_2^{B*}) \\
&< g_1'(G_1^{B*}) - (1 + r - \theta^B) g_2'(\theta^B G_1^{B*} + G_2^{B*}) < 0
\end{aligned}
$$

by using Assumption 2.1 and the proof of part (i) of Proposition 3.3. So, Eq. (22) combined with $V_{bb}(\cdot) < 0$ yields that $0 > V_b(b^{B*}, z^{B*}, \theta^A) \geq V_b(b^A, z^{B*}, \theta^A)$ for any $b^A \geq \underline{b} = b^{B*}$. Accordingly, the strict monotonicity reveals that region A must choose $b^A = b^{B*}$ and realizes (b^{B*}, z^{B*}). Noting from the proof of part (i) of Proposition 3.3 that region A is indifferent between (b^{B*}, z^{B*}) and (b^{A*}, z^{A*}), so it has no incentives to mimic region B.

Step 2 We now turn to analyze region B in a similar way. Taking the federal transfer z^{B*} as given, it solves the maximization problem: $\max_{b^B} V(b^B, z^{B*}, \theta^B)$ subject to $b^B \geq \underline{b} = b^{B*}$. The derivatives of the region's welfare function analogous to Eqs. (22) and (23) are omitted to economize on the space. Evaluating the first-order derivative at $b^B = b^{B*}$ yields $V_b(b^{B*}, z^{B*}, \theta^B) < 0$, as already shown in the proof of part (i) of Proposition 3.3. We thus have $V_b(b^B, z^{B*}, \theta^B) < 0$ for all $b^B \geq b^{B*}$ given that $V_{bb}(\cdot) < 0$. This, accordingly, implies that region B sets $b^B = b^{B*}$ and realizes (b^{B*}, z^{B*}). In addition, since applying Envelope Theorem shows that $V_z(b, z, \theta) = g_1'(\cdot) + \theta g_2'(\cdot) > 0$, $V(b^B, z^{B*}, \theta^B) > V(b^B, z^{A*}, \theta^B)$ for all feasible b^B immediately follows from part (i) of Proposition 3.3. That is, region B has no incentives to mimic region A. As a result, the redistribution scheme of the center accompanied with a lower bound imposed on public debt is incentive compatible for both regions. The proof of part (i) is hence complete.

Step 3 We now prove part (ii) in a quite similar way. Consider first the implementation in region B, and suppose it solves the maximization problem: $\max_{b^B} V(b^B, z^{A*}, \theta^B)$ subject to $b^B \leq \overline{b} \equiv b^{A*}$. Noting from the proof of part (ii) of Proposition 3.3 that

$$\begin{aligned} V_b(b^{A*}, z^{A*}, \theta^B) &= g_1'(G_1^{A*}) - (1 + r - \theta^B) g_2'(\theta^B G_1^{A*} + G_2^{A*}) \\ &> g_1'(G_1^{A*}) - (1 + r - \theta^A) g_2'(\theta^A G_1^{A*} + G_2^{A*}) > 0, \end{aligned}$$

we thus obtain from using the fact $V_{bb}(\cdot) < 0$ that $V_b(b^B, z^{A*}, \theta^B) \geq V_b(b^{A*}, z^{A*}, \theta^B) > 0$ for any feasible b^B. So, region B must choose $b^B = b^{A*}$ and realizes (b^{A*}, z^{A*}). Nevertheless, noting from the proof of part (ii) of Proposition 3.3 that region B is indifferent between (b^{B*}, z^{B*}) and (b^{A*}, z^{A*}), so it has no incentives to mimic region A. We then turn to analyze region A. Taking the federal transfer $z^{A*} > 0$ as given, it solves the maximization problem: $\max_{b^A} V(b^A, z^{A*}, \theta^A)$ subject to $b^A \leq \overline{b} = b^{A*}$. As it is easy to show that $V_b(b^A, z^{A*}, \theta^A) > 0$ for any feasible b^A, it must set $b^A = b^{A*}$ and realizes (b^{A*}, z^{A*}). As $z^{A*} > 0 > z^{B*}$ from part (ii) of Proposition 3.3, it has no incentives to mimic region B. Therefore, the redistribution scheme as well as the regional budget restriction is incentive compatible for both regions. □

Appendix B: The asymmetric-information optimum with binding participation constraints

Formally, the center solves the following maximization problem:

$$\max_{b^A, z^A, b^B, z^B} \quad V(b^A, z^A, \theta^A) + V(b^B, z^B, \theta^B)$$

subject to fiscal budget constraint (2) and these incentive-compatibility (IC) constraints and individual-rationality (IR) constraints:

$$\begin{aligned}
V(b^A, z^A, \theta^A) &\geq V(b^B, z^B, \theta^A) \quad (\mathrm{IC}_A);\\
V(b^B, z^B, \theta^B) &\geq V(b^A, z^A, \theta^B) \quad (\mathrm{IC}_B);\\
V(b^A, z^A, \theta^A) &\geq \max_{b^A} \tilde{V}(b^A, \theta^A) \equiv V_A^{\mathrm{sq}} \quad (\mathrm{IR}_A);\\
V(b^B, z^B, \theta^B) &\geq \max_{b^B} \tilde{V}(b^B, \theta^B) \equiv V_B^{\mathrm{sq}} \quad (\mathrm{IR}_B);
\end{aligned}$$

in which the superscript $^{\mathrm{sq}}$ means status quo and

$$\begin{aligned}
\tilde{V}(b^R, \theta^R) \equiv \max_{c_1^R, c_2^R} \quad & u_1(c_1^R) + g_1(y_1 + b^R - c_1^R) + u_2(c_2^R)\\
&+ g_2(\theta^R(y_1 + b^R - c_1^R) + y_2 - b^R(1+r) - c_2^R)
\end{aligned}$$

for $R \in \{A, B\}$. As such, V_A^{sq} and V_B^{sq} can be seen as two constants that are independent of the choice variables of the center.

The Lagrangian is now written as

$$\begin{aligned}
&\mathcal{L}(b^A, z^A, b^B, z^B; \mu_A, \mu_B, \xi_A, \xi_B, \lambda) = (1 + \mu_A + \xi_A)V(b^A, z^A, \theta^A)\\
&\quad -\mu_A V(b^B, z^B, \theta^A) + (1 + \mu_B + \xi_B)V(b^B, z^B, \theta^B)\\
&\quad -\mu_B V(b^A, z^A, \theta^B) - \lambda(z^A + z^B) - \xi_A V_A^{\mathrm{sq}} - \xi_B V_B^{\mathrm{sq}}, \qquad (24)
\end{aligned}$$

in which μ_A, μ_B, ξ_A, ξ_B and λ are the associated nonnegative Lagrangian multipliers. As before, we let the federal budget constraint (2) be binding so that $\lambda > 0$.

We now check to what extent the asymmetric-information optimum established in Proposition 3.3 is robust to the introduction of binding participation constraints. Indeed, the following proposition is obtained.

Proposition 5.1 *Under Assumption 2.1 and asymmetric information, the second-best optimum with binding participation constraints satisfies:*

(i) *If $\mu_A > 0$, then part (i) of Proposition 3.3 holds true whenever $\xi_B \leq \xi_A + \mu_A$.*
(ii) *If $\mu_A = 0$ and $\mu_B > 0$, then part (ii) of Proposition 3.3 holds true whenever $\xi_A \leq \xi_B + \mu_B$.*

Proof We shall complete the proof in two steps.

Step 1. Suppose IC_A is binding, then we can apply the same logic as used in the proof of Proposition 3.3 to show that IC_B must not be binding whenever $\xi_B \leq \xi_A + \mu_A$. As participation constraints are assumed to be binding, operating the maximization operator with (24) produces the following FOCs:

$$
\begin{aligned}
& b^A : g_1'(G_1^A) - (1 + r - \theta^A) g_2'(\theta^A G_1^A + G_2^A) = 0; \\
& z^A : g_1'(G_1^A) + \theta^A g_2'(\theta^A G_1^A + G_2^A) = \frac{\lambda}{1 + \mu_A + \xi_A}; \\
& b^B : g_1'(G_1^B) - (1 + r - \theta^B) g_2'(\theta^B G_1^B + G_2^B) \\
& \quad = \frac{\mu_A}{1 + \xi_B} [g_1'(G_1^B) - (1 + r - \theta^A) g_2'(\theta^A G_1^B + G_2^B)]; \\
& z^B : g_1'(G_1^B) + \theta^B g_2'(\theta^B G_1^B + G_2^B) \\
& \quad = \frac{\mu_A}{1 + \xi_B} [g_1'(G_1^B) + \theta^A g_2'(\theta^A G_1^B + G_2^B)] + \frac{\lambda}{1 + \xi_B}.
\end{aligned} \tag{25}
$$

By the first equation of FOCs (25), claim (i-a) of Proposition 3.3 immediately follows. Combining the third and fourth equations of FOCs (25), we have

$$
(1 + r) \left[g_2'(\theta^B G_1^B + G_2^B) - \frac{\mu_A}{1 + \xi_B} g_2'(\theta^A G_1^B + G_2^B) \right] = \frac{\lambda}{1 + \xi_B}, \tag{26}
$$

by which it is easy to see that $1 + \xi_B > \mu_A$. Since term $-(1 + r - \theta) g_2'(\theta G_1 + G_2)$ is strictly increasing in θ, we get from Assumption 2.1 and the third equation of FOCs (25) that

$$
\left(1 - \frac{\mu_A}{1 + \xi_B} \right) \left[g_1'(G_1^B) - (1 + r - \theta^B) g_2'(\theta^B G_1^B + G_2^B) \right] < 0,
$$

by which claim (i-b) of Proposition 3.3 immediately follows.

Moreover, by applying (26) and rearranging the first two equations of FOCs (25), we get that

$$
\begin{aligned}
g_2'(\theta^B G_1^B + G_2^B) &> \frac{\lambda}{(1 + r)(1 + \xi_B - \mu_A)} \\
&\geq \frac{\lambda}{(1 + r)(1 + \mu_A + \xi_A)} = g_2'(\theta^A G_1^A + G_2^A)
\end{aligned} \tag{27}
$$

for any $\xi_B \leq \xi_A + 2\mu_A$. Also, we can get from the second and the fourth equations of FOCs (25) that

$$
\begin{aligned}
g_1'(G_1^A) + \theta^A g_2'(\theta^A G_1^A + G_2^A) &= \frac{\lambda}{1 + \mu_A + \xi_A} \\
&\leq \frac{\lambda}{1 + \xi_B} < g_1'(G_1^B) + \theta^B g_2'(\theta^B G_1^B + G_2^B)
\end{aligned} \tag{28}
$$

for any $\xi_B \leq \xi_A + \mu_A$. Using (27) and (28), it is easy to verify that claim (i-c) of Proposition 3.3 follows from exactly the same reasoning used in "Appendix A". The proof of part (i) is thus complete.

Step 2. We now proceed to the proof of part (ii). Suppose IC_B, IR_A, IR_B are binding and IC_A is not binding, then applying these assumptions to (24) gives rise to the following FOCs:

$$\begin{aligned}
b^B &: g_1'(G_1^B) - (1+r-\theta^B)g_2'(\theta^B G_1^B + G_2^B) = 0;\\
z^B &: g_1'(G_1^B) + \theta^B g_2'(\theta^B G_1^B + G_2^B) = \frac{\lambda}{1+\mu_B+\xi_B};\\
b^A &: g_1'(G_1^A) - (1+r-\theta^A)g_2'(\theta^A G_1^A + G_2^A)\\
&= \frac{\mu_B}{1+\xi_A}[g_1'(G_1^A) - (1+r-\theta^B)g_2'(\theta^B G_1^A + G_2^A)];\\
z^A &: g_1'(G_1^A) + \theta^A g_2'(\theta^A G_1^A + G_2^A)\\
&= \frac{\mu_B}{1+\xi_A}[g_1'(G_1^A) + \theta^B g_2'(\theta^B G_1^A + G_2^A)] + \frac{\lambda}{1+\xi_A}.
\end{aligned} \tag{29}$$

Claim (ii-b) of Proposition 3.3 follows from the first equation of FOCs (29). As we have done in Step 1, using the third equation of FOCs (29) reveals that

$$\frac{\mu_B}{1+\xi_A} = \frac{g_1'(G_1^A) - (1+r-\theta^A)g_2'(\theta^A G_1^A + G_2^A)}{g_1'(G_1^A) - (1+r-\theta^B)g_2'(\theta^B G_1^A + G_2^A)} < 1$$

and

$$\left(1 - \frac{\mu_B}{1+\xi_A}\right)\left[g_1'(G_1^A) - (1+r-\theta^A)g_2'(\theta^A G_1^A + G_2^A)\right] > 0,$$

by which claim (ii-a) of Proposition 3.3 follows. Making use of the second and the fourth equations of FOCs (29), we obtain

$$\begin{aligned}
g_1'(G_1^B) + \theta^B g_2'(\theta^B G_1^B + G_2^B) &= \frac{\lambda}{1+\mu_B+\xi_B}\\
&\le \frac{\lambda}{1+\xi_A} < g_1'(G_1^A) + \theta^A g_2'(\theta^A G_1^A + G_2^A)
\end{aligned}$$

for any $\xi_A \le \mu_B + \xi_B$. The remaining proof of claim (ii-c) of Proposition 3.3 is the same as that appears in "Appendix A". The proof of part (ii) is, therefore, complete. □

Proposition 5.1 states that the asymmetric-information optimum with binding participation constraints coincide with the asymmetric-information that ignores these IR constraints up to the additional requirement that $\xi_B \le \xi_A + \mu_A$ when $\mu_A > 0$ or that $\xi_A \le \xi_B + \mu_B$ when $\mu_A = 0$ and $\mu_B > 0$. Intuitively, $\xi_B \le \xi_A + \mu_A$ means that the shadow price of region B's participation constraint is no higher than the summation of the shadow prices of region A's participation constraint and incentive-compatibility constraint. As we have shown in Proposition 3.3 that region B is the recipient while region A is the contributor of interregional redistribution in the asymmetric-information optimum under $\mu_A > 0$, other things equal, region B should be more likely to voluntarily participate in the federal redistribution arrangement, and hence $\xi_B \leq \xi_A + \mu_A$ can be interpreted as a somewhat mild restriction. Similar reasoning applies to the interpretation of $\xi_A \leq \xi_B + \mu_B$ under $\mu_A = 0$ and $\mu_B > 0$.

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